Unit 8 - Chapter 1 Test Review: Sequences and Series

1. Given an arithmetic sequence with $t_{15} = 38$ and $d = -3$.

$t_1 - 42 = 38$

$t_1 = 80$

a) Find t_1.

$t_n = t_1 + (n-1)d$

$38 = t_1 + (15-1)\cdot(-3)$

$t_{15} = 38$

$d = -3$

$t_1 + (14)\cdot(-3) = 38$

b) Which term in the sequence has the value of -295?

$t_n = t_1 + (n-1)d$

$-295 = 80 + (n-1)\cdot(-3)$

$-295 = 80 - 3n + 3$

$-3n = -295 - 83$

$-3n = -378$

$n = 126$

2 : For the arithmetic sequence: -3,2,7,12,.....

$d = 5$ $t_1 = -3$

a. Determine t_{20}.

$t_n = t_1 + (n-1)d$

$t_{20} = -3 + (20-1)\cdot 5$

$t_{20} = -3 + 95$

$t_{20} = 92$

b. Which term in the sequence has the value 212?

$t_n = t_1 + (n-1)d$

$212 = (-3) + (n-1)\cdot 5$

$-3 + 5n - 5 = 212$

$5n = 212 + 8$

$5n = 220$

$n = 44$

c. Is 318 in the sequence?

$318 = (-3) + (n-1)\cdot 5$

$-3 + 5n - 5 = 318$

$5n = 326$

$n = 65.2$

318 is not in the sequence.

3. Find the sum of the arithmetic series: 11 + 15 + 19 + … + 107

$d = 4$

$t_n = t_1 + (n-1)d$

$107 = 11 + (n-1)\cdot 4$

$11 + 4n - 4 = 107$

$4n = 100$ $n = 25$

$S_n = \frac{n(t_1 + t_n)}{2}$

$S_n = \frac{25(11+107)}{2}$

$S_n = 1475$

4. An arithmetic series has $t_1 = 5.5$ and $d = -2.5$. Determine S_{40}.

$S_n = \frac{n[2t_1 + (n-1)d]}{2}$

$S_{40} = \frac{40[2\cdot 5.5 + (40-1)\cdot(-2.5)]}{2}$

$S_{40} = 40[11 + (-97.5)]$

$S_{40} = \frac{40\cdot(-86.5)}{2}$

$S_{40} = -1730$

5. Find the first 3 terms of the arithmetic series with $t_1 = 14$, $S_n = -1207$, $t_n = -85$

$S_n = \frac{n(t_1 + t_n)}{2}$

$-1207 = \frac{n(14-85)}{2}$

$-1207 = \frac{-71n}{2}$

$-71n = -2414$

$n = 34$

$t_n = t_1 + (n-1)d$

$-85 = 14 + (34-1)d$

$33d + 14 = -85$

$33d = -99$

$d = -3$

$t_1 = 14$

$t_2 = 14 - 3 = 11$

$t_3 = 11 - 3 = 8$

6. Determine the first term of the geometric sequence $t_8 = \frac{1}{4}$ and $r = \frac{1}{4}$

$t_n = t_1 r^{n-1}$

$\frac{1}{4} = t_1 \cdot (\frac{1}{4})^{8-1}$

$t_1 \cdot (\frac{1}{4})^7 = \frac{1}{4}$

$t_1 = (\frac{1}{4})^1 \div (\frac{1}{4})^7$

$t_1 = (\frac{1}{4})^{1-7}$

$t_1 = (\frac{1}{4})^{-6}$ $t_1 = 4096$

7. Between the Canadian censuses in 2006 and 2011, the number of people who could talk in French had increased by 8%. In 2006, 97 385 people could talk in French. Assume the 5-year increase continues to be ~~7%~~ 8%. Estimate to the nearest hundred how many people will be able to talk in French in 2041.

we know: $t_1 = 97385$

$r = 1 + 8\% = 1.08$

2006 t_1, 2011 t_2, 2016 t_3, 2021 t_4, 2026 t_5, 2031 t_6, 2036 t_7, 2041 t_8

$n = 8$

$t_n = t_1 r^{n-1}$

$= 97385 \times 1.08^{8-1}$

$= 166900.78$

8. Given $t_1 = -1$ and $r = -2$,

a) Determine t_9.

$t_n = t_1 \cdot r^{n-1}$

$t_9 = (-1) \cdot (-2)^{9-1}$

$t_9 = (-1) \cdot (-2)^8$

$t_9 = (-1) \cdot 256$

$t_9 = -256$

b) The last term is -4096. How many terms are there in the sequence?

$t_n = t_1 r^{n-1}$

$-4096 = (-1) \cdot (-2)^{n-1}$

$(-2)^{n-1} = 4096$

$(-2)^{12} = 4096$

$n - 1 = 12$

$n = 13$

10. An art piece is purchased for \$380 000. Its value is increasing at 4% per year. How much will the art piece be worth after 7 years?

$t_n = t_1 \cdot r^{n-1}$

$t_n = 380000 \cdot (1 + 0.04)^{7-1}$

$t_n = \$480821$

11. Find the sum of the series $5 + 15 + 45 + \ldots + 10\,935$.

3 3

$t_n = t_1 r^{n-1}$

$10935 = 5 \cdot 3^{n-1}$

$3^{n-1} = 2187$

$3^7 = 2187$

$n - 1 = 7$

$n = 8$

$S_n = \frac{t_1(1 - r^n)}{1 - r}$

$= \frac{5(1 - 3^8)}{1 - 3}$

$= \frac{5(-6560)}{-2}$

$= \frac{-32800}{-2}$

$= 16400$

12. How many terms of the series 2 + (-4) + 8 + (-16) + . . . will yield a sum of 342?

-2 -2

$S_n = \frac{t_1(1-r^n)}{1-r}$

$342 = \frac{2[1-(-2)^n]}{1-(-2)}$

$342 = \frac{2[1-(-2)^n]}{3}$

$2[1-(-2)^n] = 1026$

$2-2(-2)^n = 1026$

$-2(-2)^n = 1024$

$(-2)^n = -512$

$(-2)^9 = -512$

$n = 9$

13. Describe what happens to term values as series continues (the first term is positive)

r>1	They move further away from the x-axis – further away from 0 (up and to the right → Increasing)
0<r<1	Moved closer to the x-axis – closer to zero (Down → decreasing)
-1<r<0	Each point moves closer to the x-axis (Alternating Sides) (Closer to zero)
r<-1	Each point moves futher from the x-axis (futher from 0) (Alternating Sides)

14. Find the sum of each geometric series.

a) 16 + 8 + 4 + . . .

$\frac{1}{2}$ $\frac{1}{2}$

$S_\infty = \frac{16}{1-\frac{1}{2}}$

$S_\infty = \frac{16}{\frac{1}{2}}$

$S_\infty = 32$

b) $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots$

$\frac{1}{3}$ $\frac{1}{3}$

$S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{3}}$

$S_\infty = \frac{\frac{1}{2}}{\frac{2}{3}}$

$S_\infty = \frac{1}{2} \cdot \frac{3}{2}$

$S_\infty = \frac{3}{4}$

c) 4 - 6 + 9 - 13.5 + . . .

$-\frac{3}{2}$ $-\frac{3}{2}$

$r = \frac{-6}{4} = -1.5$

$\because -1.5 < -1$

$\therefore$ it's divergent.

there's no sum

15. Solve

$r = \frac{3}{4}$ and $S_\infty = \frac{24}{7}$

$S_\infty = \frac{t_1}{1-r}$

$\frac{24}{7} = \frac{t_1}{1-\frac{3}{4}}$

$\frac{t_1}{\frac{1}{4}} = \frac{24}{7}$

$t_1 = \frac{24}{7} \cdot \frac{1}{4}$ $t_1 = \frac{6}{7}$

$t_1 = 21$ and $S_\infty = 63$; Find r

$S_\infty = \frac{t_1}{1-r}$

$63 = \frac{21}{1-r}$

$63(1-r) = 21$

$63 - 63r = 21$

$-63r = -42$

$r = \frac{2}{3}$

16. What is the first term and the common difference in an arithmetic series where the sum of the first 8 terms is 34 and the sum of the first 14 terms is 112.

$S_8 = 34$
$S_{14} = 112$
$t_1 = ?$
$d = ?$

$S_n = \frac{n[2t_1 + d(n-1)]}{2}$

$\begin{cases} 34 = \frac{8[2t_1 + d(8-1)]}{2} \\ 112 = \frac{14[2t_1 + d(14-1)]}{2} \end{cases}$

$\begin{cases} 8(2t_1 + 7d) = 68 \\ 14(2t_1 + 13d) = 224 \end{cases}$

$\begin{cases} 2t_1 + 7d = 8.5 \quad ① \\ 2t_1 + 13d = 16 \quad ② \end{cases}$

$① - ②$ $\quad -6d = -7.5 \quad d = 1.25 \quad r = -0.125$

17. Identify each infinite geometric series that converges. Determine the sum of any series that converges

a. 2-3+4.5-6.75+...

$-\frac{3}{2}$ $-\frac{3}{2}$

$r = -\frac{3}{2}$

$r < -1 \;\therefore$

Series Diverges

b. 1/3 +2/9 +4/27 +8/81 +...

$\frac{2}{3}$ $\frac{2}{3}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{\frac{1}{3}}{1-\frac{2}{3}}$

$= \frac{1}{3} \cdot 3 = 1$

18. Determine the sum of the geometric series below. Give the answer to 3 decimal places
-700 +350 – 175 + ... + 5.46875

$-\frac{1}{2}$

$t_n = t_1 \cdot r^{n-1}$

$5.46875 = -700 \cdot (-\frac{1}{2})^{n-1}$

$(-\frac{1}{2})^{n-1} = -\frac{1}{128}$

$(-\frac{1}{2})^7 = (-\frac{1}{128})$

$n-1 = 7$
$n = 8$

$S_n = \frac{t_1(1-r^n)}{1-r}$

$S_n = \frac{-700[1-(-\frac{1}{2})^8]}{1-(-\frac{1}{2})}$

$S_n = \frac{-700(\frac{255}{256})}{\frac{3}{2}}$

$S_n = -\frac{44625}{64} \cdot \frac{2}{3}$ (14875, 32)

$S_n \approx 464.844$

19. use the given data about each arithmetic series to determine the indicated value

a. $5 + 3\frac{1}{2} + 2 + \frac{1}{2} - -1 - \ldots$; determine the sum of 21 terms.

$n = 21 \quad t_1 = 5 \quad d = -\frac{3}{2} \quad S_{21} = ?$

$S_n = \frac{n[2t_1 + d(n-1)]}{2}$

$S_{21} = \frac{21[10 + (-\frac{3}{2}) \cdot 20]}{2}$

$S_{21} = \frac{21[10 + (-30)]}{2}$

$S_{21} = -210$

b. $S_{12} = 78$ and $t_1 = -21$; determine t_{12}

$S_n = \frac{n(t_1 + t_n)}{2}$

$78 = \frac{12(-21 + t_{12})}{2}$

$12(-21 + t_{12}) = 156$

$(-21 + t_{12}) = 13$

$t_{12} = 13 + 21$

$t_{12} = 34$

20. a. for the infinite geometric series below identify which series converges and which series diverges. Justify you answer.

i. 100-150+225-337.5 +…..

$t_1 = 100$ $r = -\frac{3}{2}$ diverges.

ii. 10+5+2.5+1.25

$t_1 = 10$ $r = \frac{1}{2}$ converges

b. For which series in part a can you determine the sum? Find the sum.

ii 18.75

21. For the arithmetic sequence given, find the following: 1,4,7,10,...

$d=3$

a. t_8

$t_n = t_1 + (n-1)d$

$t_8 = 1 + (8-1)\cdot 3$

$t_8 = 1 + 21$

$t_8 = 22$

b. t_{20}

$t_n = t_1 + (n-1)d$

$t_{20} = 1 + 19\cdot 3$

$t_{20} = 1 + 57$

$t_{20} = 58$

c. t_n (general term)

$t_n = t_1 + (n-1)d$

$t_n = 1 + 3(n-1)$

22. Determine the number of terms in the arithmetic sequences.

$n = ?$

a. 3,5,7,…129

$t_1 = 3$ $t_n = 129$ $d = 2$

$t_n = t_1 + (n-1)d$

$129 = 3 + (n-1)2$

$2n - 2 + 3 = 129$

$2n = 128$ $n = 64$

b. -29,-24,-19,…126

$t_1 = -29$ $t_n = 126$ $d = 5$

$t_n = t_1 + (n-1)d$

$126 = -29 + (n-1)5$

$5n - 5 - 29 = 126$

$5n = 160$ $n = 32$

23. Determine the common difference and the first term of the arithmetic sequence given the following:

$t_5 = 16$ and $t_8 = 25$

$t_5 = 16$ $t_8 = 25$ $t_1 = ?$ $d = ?$

$t_n = t_1 + (n-1)d$

$$\begin{cases} 16 = t_1 + (5-1)d \\ 25 = t_1 + (8-1)d \end{cases}$$

$$\begin{cases} 16 = t_1 + 4d & ① \\ 25 = t_1 + 7d & ② \end{cases}$$

① − ②

$-9 = -3d$

$d = 3$

$12 + t_1 = 16$

$t_1 = 4$

24. A museum purchases a painting for $15,000. The painting increases in value each year by 10% of the original price. What is the value of the painting after ten years?

$t_1 = 15000 \quad r = 1+10\% = 1.1 \quad n = 10$

$t_n = t_1 \cdot r^{n-1}$

$= 15000 \cdot 1.1^{10-1} \approx \$35369$

25. Find the sum of the first ten terms of each arithmetic series.

a. 2+8+14+...

$d=6 \quad n=10 \quad t_1=2$

$S_n = \frac{n[2t_1 + d(n-1)]}{2}$

$S_{10} = \frac{10[4+6(10-1)]}{2}$

$= 5(4+54) = 290$

b. 45+39+33+...

$d=-6 \quad t_1=45 \quad n=10$

$S_n = \frac{n[2t_1 + d(n-1)]}{2}$

$S_{10} = \frac{10[90+(-6)\cdot 9]}{2}$

$= 5(90-54) = 180$

26. Find the sum of the arithmetic series. 20+14+8+...-70

$t_1 = 20 \quad d=-6 \quad t_n = -70$

$t_n = t_1 + (n-1)d$

$-70 = 20 + (n-1)(-6)$

$-6n + 6 + 20 = -70 \quad n = 16$

$S_n = \frac{n(t_1+t_n)}{2}$

$= \frac{16(20-70)}{2}$

$= -400$

27. For the geometric sequence 1,3,9,27,... find the following:

$t_1 = 1 \quad r = 3$

a. t_{11}

$t_n = t_1 \cdot r^{n-1}$

$t_{11} = 1 \cdot 3^{11-1}$

$t_{11} = 3^{10}$

$t_{11} = 59049$

b. t_{20}

$t_n = t_1 \cdot r^{n-1}$

$t_{20} = 1 \cdot 3^{20-1}$

$t_{20} = 1 \cdot 3^{19}$

$t_{20} = 1162261467$

28. Find the number of terms, "n", in each of the geometric sequences.

a. 3,6,12,...,1536

$t_1 = 3 \quad r = 2 \quad t_n = 1536 \quad n = ?$

$t_n = t_1 \cdot r^{n-1}$

$1536 = 3 \cdot 2^{n-1}$

$2^{n-1} = 512$

$2^9 = 512$

$n-1 = 9$

$n = 10$

b. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2048}$

$t_1 = \frac{1}{2} \quad r = \frac{1}{2} \quad t_n = \frac{1}{2048}$

$t_n = t_1 \cdot r^{n-1}$

$\frac{1}{2048} = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^{n-1} = \frac{1}{1024}$

$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

$n-1 = 10$

$n = 11$

29. In a geometric sequence $t_1 = 3$ and $t_5 = 243$. Find the common ratio, "r", and the terms between t_1 and t_5.

$t_1 = 3 \quad t_5 = 243 \quad r = ? \quad t_1 = ? \quad t_2 = ? \quad t_3 = ? \quad t_4 = ? \quad t_5 = ?$

$t_n = t_1 r^{n-1}$

$243 = 3 \cdot r^{5-1}$

$3r^4 = 243$

$r^4 = 81$

$3^4 = 81$

$r = 3$

$t_1 = 3$

$t_2 = 3 \cdot 3 = 9$

$t_3 = 9 \cdot 3 = 27$

$t_4 = 27 \cdot 3 = 81$

$t_5 = 81 \cdot 3 = 243$

30. For each geometric series, determine the sum indicated.

a. $2+8+32+\ldots$ (S_8)

$t_1 = 2 \quad r = 4 \quad n = 8$

$S_n = \dfrac{t_1(1-r^n)}{1-r}$

$S_8 = \dfrac{2(1-4^8)}{1-4}$

$S_8 = \dfrac{2(1-65536)}{-3}$

$S_8 = \dfrac{-131070}{-3}$

$S_8 = 43690$

b. $24-12+6-\ldots$ (S_{10})

$t_1 = 24 \quad r = -\frac{1}{2} \quad n = 10$

$S_n = \dfrac{t_1(1-r^n)}{1-r}$

$S_{10} = \dfrac{24 \cdot [1-(-\frac{1}{2})^{10}]}{\frac{3}{2}}$

$S_{10} = \dfrac{24 \cdot [1-\frac{1}{1024}]}{\frac{3}{2}}$

$S_{10} = \dfrac{1023}{64}$

31. Calculate the sum of the geometric series. $960+480+240+\ldots+15$

$\frac{1}{2}$

$t_1 = 960 \quad r = \frac{1}{2} \quad t_n = 15$

$S_n = \dfrac{t_1(1-r^n)}{1-r}$

$= \dfrac{960(1-\frac{1}{2}^7)}{1-\frac{1}{2}} = 1905$

$t_n = t_1 \cdot r^{n-1}$

$15 = 960 \cdot (\frac{1}{2})^{n-1}$

$(\frac{1}{2})^{n-1} = \frac{1}{64}$

$(\frac{1}{2})^6 = \frac{1}{64}$

$n-1 = 6$

$n = 7$

32. A doctor prescribes 200mg of medication on the first day of treatment. The dosage is reduced by one half on each successive day for one week. What is the total amount of medication prescribed, to the nearest milligram?

we know: $t_1 = 200$

$r = \frac{1}{2}$

$n = 7$

$S_n = \dfrac{t_1(1-r^n)}{1-r} = \dfrac{200(1-\frac{1}{2}^7)}{1-\frac{1}{2}} \approx 397\text{ mg}$

∴ the total amount of medication is 397 mg

33. State whether each infinite geometric series is **convergent or divergent**. **State the sum** of the series, **if there is a sum.**

a. $1-4+16-64+\ldots$

$-4 \quad -4$

$t_1 = 1 \quad r = -4$

$r < -1$

∴ Divergent

b. $12+3+\frac{3}{4}+\frac{3}{16}+\ldots$

$\frac{1}{4} \quad \frac{1}{4}$

$t_1 = 12 \quad r = \frac{1}{4}$

$S_\infty = \dfrac{t_1}{1-r}$

$= \dfrac{12}{1-\frac{1}{4}}$

$= 12 \cdot \frac{4}{3}$

$= 16$

convergent